4. Dwumian Newtona i wzory skróconego mnożenia – odpowiedzi i wskazówki

Symbol Newtoma i wzór Newtona

4.1. **a)** 1, **b)** 10, **c)** 1, **d)** 435.

Wskazówka: **a)** $\binom{3}{3}=\frac{3!}{3!\cdot(3-3)!}=\frac{1!}{0!}=1$, **c)** $\binom{10}{0}=\frac{10!}{0!\cdot(10-0)!}=\frac{10!}{10!}=1$,

d) $\binom{30}{23}=\frac{30!}{2!\cdot(30-2)!}=\frac{28!\cdot 29\cdot 30}{28!\cdot 2}=870$.

4.4. $(a+b)^3=\binom{3}{0}a^3b^0+\binom{3}{1}a^2b+\binom{3}{2}ab^2+\binom{3}{3}b^3$.

4.6. **a)** $120\sqrt{3}+208$, **b)** $8\left(239\sqrt{2}-338\right)$, **c)** $369\sqrt{3}+18\sqrt{6}$.

4.7. $\binom{5}{2}\left(\sqrt[3]{3}\right)^3\cdot\left(\sqrt{2}\right)^2=60$.

4.8. 180.

Wzory skróconego mnożenia

4.9. **a)** $11\left(x^2+3y^2\right)$, **b)** $5-2x^3$, **c)** $22x$, **d)** $3x^3+2y^3$.

4.10. **a)** $4x^3-6x$, $6\sqrt{3}$, **b)** $8\sqrt{3}x\left(x^2+3\right)$, 144,

c) $6\sqrt{2}x^2+4\sqrt{2}$, $22\sqrt{2}$, **d)** $2x\left(x^2+15\right)$, $36\sqrt{3}$.

4.11. **a)** $(a-b+c)(a+b-c)$, **b)** $3x(2x+1)$,

c) $\left(\sqrt{a}-3\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$, gdy $a\geq 0$ i $b\geq 0$,

d) $x(3x-2y-1)(3x+2y-1)$,

e) $(x-a)(x+a)(2a-x)$,

f) $(2m-1)\left(64m^6+32m^5+16m^4+8m^3+4m^2+2m+1\right)$,

g) $(1-3x)\left(1+3x+9x^2+27x^3+81x^4\right)$

Nr zadania	**4.12. a)**	**4.12. b)**	**4.12. c)**
Odpowiedzi	–144	44	16

4.13. a) $7(x-1)(1-3x)$, **b)** $x(x+2)\left(x^2+2x+2\right)$,

c) $(x-1)(x+1)\left(x^2+x+1\right)\left(x^2-x+1\right)\left(x^2+1\right)\left(x^2-\sqrt{3}x+1\right)\left(x^2+\sqrt{3}x+1\right)$.

Wskazówka: **a)** $(2x-3)^2-(4-5x)^2=[(2x-3)-(4-5x)][(2x-3)+(4-5x)]=(7x-7)(1-3x)$.

b) $(x+1)^4-1=\left[(x+1)^2-1\right]\left[(x+1)^2+1\right]=(x+1-1)(x+1-1)\left(x^2+2x+2\right)=x(x+2)\left(x^2+2x+2\right)$.

c) $x^{12}-1=\left(x^6-1\right)\left(x^6+1\right)=\left(x^3-1\right)\left(x^3+1\right)\left(x^6+1\right)=$ $\quad x^6+1=\left(x^2\right)^3+1^3$

$=(x-1)\left(x^2+x+1\right)(x+1)\left(x^2-x+1\right)\left(x^2+1\right)\left(x^4-x^2+1\right)$.

Zatem $x^{12}-1=(x-1)\left(x^2+x+1\right)(x+1)\left(x^2-x+1\right)\left(x^2+1\right)\left(x^1-\sqrt{3}x+1\right)\left(x^2+\sqrt{3}x+1\right)$.

4.14. a) $(x+1)^2(x-1)$, **b)** $\left(\sqrt{3}-1\right)\left(\sqrt{2}+2\right)$.

Wskazówka: **a)** $x^3+x^2-x-1=$

$=x^2(x+1)-1\cdot(x+1)=$

$=(x+1)\left(x^2-1\right)=$

$=(x+1)(x+1)(x-1)=(x+1)^2(x-1)$.

Wskazówka: **b)** $\sqrt{6}-\sqrt{2}+2\sqrt{3}-2=\sqrt{2}\cdot\sqrt{3}-\sqrt{2}+2\sqrt{3}-2=$

$=\sqrt{2}\left(\sqrt{3}-1\right)+2\left(\sqrt{3}-1\right)=\left(\sqrt{3}-1\right)\left(\sqrt{2}+2\right)$.

4.15. a) $\sqrt{3}+\sqrt{2}$, **b)** $\sqrt[3]{9}+2\sqrt[3]{3}+4$,

c) $2\sqrt{2}-\sqrt{3}-\sqrt{6}+2$, **d)** $-2\left(\sqrt{6}+\sqrt{2}-2\sqrt{3}-2\right)$.

Wskazówka: **c)** $\dfrac{1}{\left(\sqrt{2}-1\right)\left(\sqrt{3}+2\right)}\cdot\dfrac{\left(\sqrt{2}+1\right)\left(\sqrt{3}-2\right)}{\left(\sqrt{2}+1\right)\left(\sqrt{3}-2\right)}=\dfrac{\left(\sqrt{2}+1\right)\left(\sqrt{3}-2\right)}{\left[\left(\sqrt{2}\right)^2-1^2\right]\left[\left(\sqrt{3}\right)^2-2^2\right]}=-\sqrt{6}+2\sqrt{2}-\sqrt{3}+2$,

Wskazówka: **d)** $\dfrac{8}{\sqrt{6}-\sqrt{2}+2\sqrt{3}-2}=\dfrac{8}{\sqrt{2}\cdot\sqrt{3}-\sqrt{2}+2\sqrt{3}-2}=\dfrac{8}{\sqrt{2}\left(\sqrt{3}-1\right)+2\left(\sqrt{3}-1\right)}=$

$=\dfrac{8}{\left(\sqrt{3}-1\right)\left(\sqrt{2}+2\right)}\cdot\dfrac{\left(\sqrt{3}+1\right)\left(\sqrt{2}-2\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{2}-2\right)}=-2\left(\sqrt{6}+\sqrt{2}-2\sqrt{3}-2\right)$.

4.16. a) 23, **b)** –2, **c)** $39\frac{1}{2}$, **d)** 14.

Wskazówka: **a)** $x+\frac{1}{x}=5$, czyli $\left(x+\frac{1}{x}\right)^2=5^2$, skąd $x^2+\frac{1}{x^2}=23$,

b) $x+\frac{1}{x}=-2$, czyli $\left(x+\frac{1}{x}\right)^3=(-2)^3$, więc $x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=-8$, skąd $x^3+\frac{1}{x^3}=-2$,

c) jeżeli $x^2+y^2=(x+y)^2-2xy=8$, to $xy=-\frac{7}{2}$.

$x^4+y^4=\left(x^2+y^2\right)^2-2x^2y^2=8^2-2\cdot\left(-\frac{7}{2}\right)^2=64-\frac{49}{2}=39{,}5$,

d) $x^3+\frac{1}{x^3}=52$, czyli $\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=52$. Jeżeli $x+\frac{1}{x}=t$, to równanie ma postać $t^3-3t-52=0$,

skąd $t=4$, czyli $x+\frac{1}{x}=4$. Jeżeli $x+\frac{1}{x}=4$, to $\left(x+\frac{1}{x}\right)^2=16$, skąd $x^2+\frac{1}{x^2}=14$.

4.17. a) $xy=-\frac{9}{2}$, **b)** $xy=3$, **c)** $xy=-2$.

Wskazówka: **a)** $x^2+y^2=(x+y)^2-2xy$,

b) $x^3+y^3=(x+y)^3-3xy(x+y)$, **c)** $x^3-y^3=(x-y)^3-3xy(x-y)$.

4.18. a) $(x-3)(x-2)(x+2)$, **b)** $\left(x-\sqrt{5}\right)(x-2)\left(x+\sqrt{5}\right)$,

c) $(3x-2)(3x+2)\left(3x^2+4\right)$, **d)** $(x-3)(x+3)\left(x^2+3x+9\right)\left(x^2-3x+9\right)$,

e) $\left(x^2-x\sqrt{10}+5\right)\left(x^2+x\sqrt{10}+5\right)$, **f)** $(x-5)(x-1)(x+6)$,

g) $5\left(x-\frac{1}{5}\right)(x+5)\left(x^2+1\right)$, **h)** $x(x-1)(x+1)(x-2)(x+2)(x-3)(x+3)$.

4.19. *Wskazówka:*

a) $63^3+17^3=(63+17)\left(63^2-63\cdot17+17^2\right)=8\cdot10\left(63^2-63\cdot17+17^2\right)$,

b) $9^{16}-17^{16}\left(9^8\right)^2-\left(7^8\right)^2=\left(9^8-7^8\right)\left(9^8+7^8\right)=\left(9^4-7^4\right)\left(9^4+7^4\right)\left(9^8+7^8\right)=$

$=\left(9^2-7^2\right)\left(9^2+7^2\right)\left(9^4+7^4\right)\left(9^8+7^8\right)=13\cdot10\cdot\left(9^2-7^2\right)\left(9^4+7^4\right)\left(9^8+7^8\right)$,

c) $3^{12}-2^6=\left(3^4\right)^3-\left(2^2\right)^3=\left(3^4-2^2\right)\left(\left(3^4\right)^2+3^4\cdot2^2+\left(2^2\right)^2\right)=77\left(3^8+81\cdot4+19\right)$,

d) $9^n+5^n-2=\left(9^n-1\right)+\left(5^n-1\right)=(9-1)\left(9^{n-1}+9^{n-2}+\ldots+9+1\right)+(5-1)\left(5^{n-1}+5^{n-2}+\ldots+5+1\right)=$

$=4\left[2\left(9^{n-1}+9^{n-2}+\ldots+9+1\right)+5^{n-1}+5^{n-2}+\ldots+5+1\right]$.